

16-th Nordic Mathematical Contest

April 4, 2002

1. A trapezoid $ABCD$ with $AB \parallel CD$ and $AD < CD$ is inscribed in a circle c . Let DP be a chord parallel to AC . The tangent to c at D meets the line AB at E , and the lines PB and DC meet at Q . Prove that $EQ = AC$.
2. Let N balls, numbered 1 through N , be distributed over two urns. A ball is taken from one urn and moved to the other. It turns out that the arithmetic means of the numbers on the balls in each urn increased by the same number x . What is the greatest possible value of x ?
3. Let $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n$ be real numbers with a_1, \dots, a_n distinct. Show that if the product $(a_i + b_1)(a_i + b_2) \cdots (a_i + b_n)$ takes the same value for every $i = 1, 2, \dots, n$, then the product $(a_1 + b_j)(a_2 + b_j) \cdots (a_n + b_j)$ also takes the same value for every $j = 1, 2, \dots, n$.
4. Eva, Per, and Anna randomly select different nine-digit integers made of digits $1, 2, \dots, 9$ and check if they are divisible by 11. Anna claims that the probability that the number is divisible by 11 is exactly $1/11$; Eva believes that this probability is less than $1/11$, while Per thinks that it is greater than $1/11$. Who of them is right?