12-th Nordic Mathematical Contest

April 2, 1998

1. Find all functions $f : \mathbb{Q} \to \mathbb{Q}$ such that
$$f(x+y)+f(x-y)=2f(x)+2f(y) \quad \text{for all } x,y \in \mathbb{Q}.$$

2. Two circles C_1 and C_2 with centers M_1 and M_2 respectively intersect at A and B. A point P is taken on the segment AB so that $AP \neq BP$. The line through P perpendicular to M_1P intersects C_1 at C and D, and the line through P perpendicular to M_2P intersects C_2 at E and F. Prove that C,D,E,F are the vertices of a rectangle.

3. (a) For which positive integers n is there a permutation $x_1,\dots,x_n$ of $1,2,\dots,n$ such that k divides $x_1+\cdots+x_k$ for $k=1,\dots,n$?

   (b) Does there exist an infinite sequence $x_1,x_2,\dots$ containing every positive integer exactly once such that k divides $x_1+\cdots+x_k$ for every $k \in \mathbb{N}$?

4. Let n be a positive integer. Prove that the number of odd numbers among the binomial coefficients $\binom{n}{0},\binom{n}{1},\dots,\binom{n}{n}$ is a power of two.

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