11-th Nordic Mathematical Contest

March 1997

1. For any set A of positive integers, let n_A be the number of triples (x,y,z) of elements of A with $x<y$ and $x+y=z$. If A is a seven-element set, find the maximum possible value of n_A.

2. Assume there is a point P inside a convex quadrilateral $ABCD$ such that the triangles ABP, BCP, CDP, DAP have the same area. Prove that one of the diagonals of $ABCD$ bisects the other.

3. Points A,B,C,D in the plane are such that three of the segments AB, AC, AD, BC, BD, CD have length a, whereas the other three have length $b>a$. Find all possible values of the ratio b/a.

4. A function $f:\mathbb{N}_0\to\mathbb{N}$ satisfies for all x

   (i) $f(2x)=2f(x)$,

   (ii) $f(4x+1)=4f(x)+3$,

   (iii) $f(4x-1)=2f(2x-1)-1$.

   Prove that f is injective.

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