10-th Nordic Mathematical Contest

March 11, 1996

1. Show that there exists a positive multiple of 1996 whose sum of digits is 1996.

2. Find all real numbers x such that $x^n + x^{-n}$ is an integer for all $n \in \mathbb{Z}$.

3. The circle with the altitude from A in a triangle ABC as a diameter intersects AB at D and AC at E. Prove that the circumcenter of $\triangle ABC$ lies on the line containing the altitude from A in triangle ADE.

4. Given a positive integer a, a function f from $\mathbb{N}$ to $\mathbb{R}$ satisfies $f(a) = f(1995)$, $f(a+1) = f(1996)$, $f(a+2) = f(1997)$ and

$$f(n+a) = \frac{f(n)-1}{f(n)+1} \quad \text{for all } n \in \mathbb{N}.$$

   (a) Prove that $f(n+4a) = f(n)$ for all n.

   (b) Determine the smallest possible value of a.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com