9-th Nordic Mathematical Contest

March 15, 1995

1. Let AB be the diameter of a semicircle with center O and let C be a point on the semicircle such that OC is perpendicular to AB. Let P be an arbitrary point on the arc BC. The lines CP and AB meet at Q. Point Q is taken on AP so that QR is orthogonal to AB. Show that $BQ = QR$.

2. Messages are coded as sequences of zeros and ones. Only sequences with not more than two consecutive zeros or ones are allowed. How many permitted 12-digit sequences are there?

3. Let $x_1, x_2, \dots, x_n$ be $n \geq 2$ real numbers with $x_1 + \cdots + x_n \geq 0$ and $x_1^2 + \cdots + x_n^2 = 1$. If $\max\{x_1, \dots, x_n\} = M$, prove that
$$M \geq \frac{1}{\sqrt{n(n-1)}}.$$
Decide if equality is possible.

4. Show that there are infinitely many non-congruent triangles T such that
   (a) the side lengths of T are consecutive integers, and
   (b) the area of T is an integer.

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