2-nd Nordic Mathematical Contest

April 11, 1988

1. A positive integer n has the following property: If three last digits of n are removed, the remaining number is $\sqrt[3]{n}$. Find n.

2. Let a, b and c be real numbers different from 0 and $a \geq b \geq c$. Prove that inequality
$$\frac{a^3 - c^3}{3} \geq abc\left(\frac{a-b}{c} + \frac{b-c}{a}\right)$$
holds. When does the equality hold?

3. Two spheres with the same center have radiuses r and R, where $r < R$. From the surface of the bigger sphere we'll try to select three points A, B and C such that all sides of the triangle ABC coincide with the surface of the smaller sphere. Prove that this selection is possible if and only if $R \leq 2r$.

4. Let m_n be a smallest value of the function
$$f_n(x) = \sum_{k=0}^{2n} x^k.$$
Prove that $m_n \to \frac{1}{2}$, when $n \to \infty$.

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