13-th Mexican Mathematical Olympiad 1999.

Oaxaca, Oaxaca

First Day

1. On a table there are 1999 counters, red on one side and black on the other side, arranged arbitrarily. Two people alternately make moves, where each move is of one of the following two types:

   (1) Remove several counters which all have the same color up;

   (2) Reverse several counters which all have the same color up.

   The player who takes the last counter wins. Decide which of the two players (the one playing first or the other one) has a wining strategy.

2. Prove that there are no 1999 primes in an arithmetic progression that are all less than 12345.

3. A point P is given inside a triangle ABC. Let D,E,F be the midpoints of AP,BP,CP, and let L,M,N be the intersection points of BF and CE, AF and CD, AE and BD, respectively.

   (a) Prove that the area of hexagon $DNELFM$ is equal to one third of the area of triangle ABC.

   (b) Prove that DL,EM, and FN are concurrent.

Second Day

4. An 8×8 board is divided into unit squares. Ten of these squares have their centers marked. Prove that either there exist two marked points on the distance at most $\sqrt{2}$, or there is a point on the distance $1/2$ from the edge of the board.

5. In a quadrilateral $ABCD$ with $AB \parallel CD$, the external bisectors of the angles at B and C meet at P, while the external bisectors of the angles at A and D meet at Q. Prove that the length of PQ equals the semiperimeter of $ABCD$.

6. A polygon is called *orthogonal* if all its sides have integer lengths and any two consecutive sides are perpendicular. Prove that if an orthogonal polygon can be cut into 2×1 rectangles, then it has at least one side of even length.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com