3-rd Mexican Mathematical Olympiad 1989.

Metepec, Puebla

First Day

1. In a triangle ABC the area is 18, the length AB is 5, and the medians from A and B are orthogonal. Find the lengths of the sides BC, AC.

2. Find two positive integers a, b such that

$$a \mid b^2,\ b^2 \mid a^3,\ a^3 \mid b^4,\ b^4 \mid a^5,\ \text{ but } a^5 \nmid b^6.$$

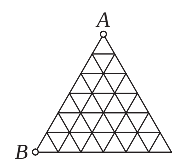
3. Prove that there is no 1989-digit natural number at least three of whose digits are equal to 5 and such that the product of its digits equals their sum.

Second Day

1. Find the smallest possible natural number $n = \overline{a_m \dots a_2 a_1 a_0}$ (in decimal system) such that the number $r = \overline{a_1 a_0 a_m \dots a_2 0}$ equals $2n$.

2. Let C_1 and C_2 be two tangent unit circles inside a circle C of radius 2. Circle C_3 inside C is tangent to the circles C, C_1, C_2, and circle C_4 inside C is tangent to C, C_1, C_3. Prove that the centers of C_1, C_2, C_3 and C_4 are vertices of a rectangle.

3. Determine the number of paths from A to B on the picture that go along gridlines only, do not pass through any point twice, and never go upwards?



The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com