1-st Mexican Mathematical Olympiad 1987.

Xalapa, Veracruz

First Day

1. Prove that if the sum of two irreducible fractions is an integer then the two fractions have the same denominator.

2. How many positive divisors does number 20! have?

3. Consider two lines l and l' and a fixed point P equidistant from these lines. What is the locus of projections M of P on AB, where A is on l, B on l', and triangle APB is right?

4. Calculate the product of all positive integers less than 100 and having exactly three positive divisors. Show that this product is a square.

Second Day

5. In a right triangle ABC, M is a point on the hypotenuse BC and P and Q the projections of M on AB and AC respectively. Prove that for no such point M do the triangles BPM, MQC and the rectangle $AQMP$ have the same area.

6. Prove that for every positive integer n the number $(n^3-n)(5^{8n+4}+3^{4n+2})$ is a multiple of 3804.

7. Show that the fraction $\frac{n^2+n-1}{n^2+2n}$ is irreducible for every positive integer n.

8. (a) Three lines l,m,n in space pass through point S. A plane perpendicular to m intersects l,m,n at A,B,C respectively. Suppose that $\angle ASB = \angle BSC = 45^\circ$ and $\angle ABC = 90^\circ$. Compute $\angle ASC$.

   (b) Furthermore, if a plane perpendicular to l intersects l,m,n at P,Q,R respectively and $SP = 1$, find the sides of triangle PQR.

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