1. Let $F = \{1, 2, \ldots, 100\}$ and let $G$ be any 10-element subset of $F$. Prove that there exist two disjoint nonempty subsets $S$ and $T$ of $G$ with the same sum of elements.

2. Suppose that in the exterior of a convex quadrilateral $ABCD$ equilateral triangles $XAB, YBC, ZCD, WDA$ with centroids $S_1, S_2, S_3, S_4$ respectively are constructed. Prove that $S_1S_3 \perp S_2S_4$ if and only if $AC \perp BD$.

3. Let $c_1, \ldots, c_n, b_1, \ldots, b_n \ (n \geq 2)$ be positive real numbers. Prove that the equation

$$\sum_{i=1}^{n} c_i \sqrt{x_i - b_i} = \frac{1}{2} \sum_{i=1}^{n} x_i$$

has a unique solution $(x_1, \ldots, x_n)$ if and only if $\sum_{i=1}^{n} c_i^2 = \sum_{i=1}^{n} b_i$.

4. Let $P, Q, R, S$ be the midpoints of the sides $BC, CD, DA, AB$ of a convex quadrilateral, respectively. Prove that

$$4(AP^2 + BQ^2 + CR^2 + DS^2) \leq 5(AB^2 + BC^2 + CD^2 + DA^2).$$