9-th Korean Mathematical Olympiad 1996

Final Round

First Day – April 13, 1996.

1. Find the largest integer n such that one cannot divide a square into n smaller squares. Justify your answer.

2. Find the smallest integer $n \geq 1996$ for which all coefficients in the expansion of $(x+y)^n$ are odd.

3. Let l be a line having no common points with a triangle ABC. Let L, M, N be the projections of A, B, C onto l, and let LX, MY, NZ be the perpendiculars from L, M, N to BC, CA, AB, respectively. Prove that these three perpendiculars are concurrent.

Second Day – April 14, 1996.

4. Four-digit natural numbers A, B can be written as $A = \overline{abcd}$ and $B = \overline{dcba}$, where a, b, c, d are (decimal) digits and a and d are nonzero. If $p = \gcd(A, B)$ is prime and $B = (A+2)p$, find all possible values of A.

5. Two radii OA and OB of a unit circle form an angle α, $0 < \alpha < \pi/2$. Let P be an arbitrary point on the arc AB. A ray of light from P is reflected from the segments OB, OA and the arc AB so that it moves along the sides of a fixed triangle PQR, with Q on OB and R on OA. Prove that the perimeter of $\triangle PQR$ does not depend on P, and find it.

6. A function $f : \mathbb{R} \to \mathbb{R}$ satisfies the following two conditions:

   (i) $f(0) = 0$, $f(1) = 1$;

   (ii) $f\left(x^2 + \frac{1}{x}\right) = f(x)^2 + f\left(\frac{1}{x}\right)$ for all $x \neq 0$.

   Show that f is not bounded from above.

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