6-th Korean Mathematical Olympiad 1993

Final Round

First Day – April 17, 1993.

1. Consider a 9×9 array of white squares. Find the largest $n \in \mathbb{N}$ with the property: No matter how one chooses n out of 81 white squares and color in black, there always remains a 1×4 array of white squares (either vertical or horizontal).

2. Let be given a triangle ABC with $BC = a$, $CA = b$, $AB = c$. Find point P in the plane for which $aAP^2 + bBP^2 + cCP^2$ is minimum, and compute this minimum.

3. Find the smallest $x \in \mathbb{N}$ for which $\dfrac{7x^{25} - 10}{83}$ is an integer.

Second Day – April 18, 1993.

4. An integer which is the area of a right-angled triangle with integer sides is called *Pythagorean*. Prove that for every positive integer $n > 12$ there exists a Pythagorean number between n and $2n$.

5. Given $n \in \mathbb{N}$, find all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that for all x,
$$\binom{n}{0} f(x) + \binom{n}{1} f\left(x^2\right) + \binom{n}{2} f\left(x^{2^2}\right) + \cdots + \binom{n}{n} f\left(x^{2^n}\right) = 0.$$

6. Consider a triangle ABC with $BC = a$, $CA = b$, $AB = c$. Let D be the midpoint of BC and E be the intersection of the bisector of $\angle A$ with BC. The circle through A, D, E meets AC, AB again at F, G respectively. Let $H \neq B$ be a point on AB with $BG = GH$. Prove that triangles EBH and ABC are similar and find the ratio of their areas.

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