6-th Korean Mathematical Olympiad 1993

Final Round

First Day – April 17, 1993.

1. Consider a $9 \times 9$ array of white squares. Find the largest $n \in \mathbb{N}$ with the property:
   No matter how one chooses $n$ out of 81 white squares and color in black, there always remains a $1 \times 4$ array of white squares (either vertical or horizontal).

2. Let be given a triangle $ABC$ with $BC = a$, $CA = b$, $AB = c$. Find point $P$ in the plane for which $aAP^2 + bBP^2 + cCP^2$ is minimum, and compute this minimum.

3. Find the smallest $x \in \mathbb{N}$ for which $\frac{7x^{25} - 10}{83}$ is an integer.


4. An integer which is the area of a right-angled triangle with integer sides is called Pythagorean. Prove that for every positive integer $n > 12$ there exists a Pythagorean number between $n$ and $2n$.

5. Given $n \in \mathbb{N}$, find all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that for all $x$,
   \[
   \binom{n}{0} f(x) + \binom{n}{1} f(x^2) + \binom{n}{2} f(x^4) + \cdots + \binom{n}{n} f(x^{2^n}) = 0.
   \]

6. Consider a triangle $ABC$ with $BC = a$, $CA = b$, $AB = c$. Let $D$ be the midpoint of $BC$ and $E$ be the intersection of the bisector of $\angle A$ with $BC$. The circle through $A, D, E$ meets $AC, AB$ again at $F, G$ respectively. Let $H \neq B$ be a point on $AB$ with $BG = GH$. Prove that triangles $EBH$ and $ABC$ are similar and find the ratio of their areas.