1. Consider the polynomial

\[ f(x) = (1 + x)^{n_1} + (1 + x)^{n_2} + \cdots + (1 + x)^{n_k}, \]

where \( n_1, n_2, \ldots, n_k \) are positive integers. For each positive integer \( a \), find the minimum of the coefficient at \( x^a \) as the \( k \)-tuple \( (n_1, \ldots, n_k) \) varies over all \( k \)-tuples with sum \( n \).

2. For all natural numbers \( m, n \), prove that

\[
\frac{1}{\sqrt{n} + 1} + \frac{1}{\sqrt{m} + 1} \geq 1.
\]

3. Equilateral triangles \( ABM \) and \( CDP \) are drawn outside a convex quadrilateral \( ABCD \), and equilateral triangles \( BCN \) and \( DAQ \) are drawn inside \( ABCD \). Suppose that among the points \( M, N, P, Q \), no three are on a line. Prove that \( MNPQ \) is a parallelogram.

4. On the coordinate plane, a piece can move 1 right or 1 up per move. Find the number of possible ways for the piece to move from point \((0,0)\) to \((n,n)\), not passing through any of the points \((1,1), (2,2), \ldots, (n-1,n-1)\).

5. Consider the matrix \( A = (a_{ij})_{i,j=1}^{1999} \) given by \( a_{ij} = 1 \) if \( i \geq j \) and \( a_{ij} = 0 \) otherwise. Find the number of ways of choosing 1998 ones from the matrix so that no two of them are in the same row or column.

6. Let \( a, b, c \) be positive real numbers with \( abc \geq 1 \). Prove that

\[
\frac{1}{a + b^4 + c^4} + \frac{1}{a^4 + b + c^4} + \frac{1}{a^4 + b^4 + c} \leq 1.
\]

7. If \( I \) is the incenter of a triangle \( ABC \), prove that

\[
IA^2 + IB^2 + IC^2 \geq \frac{AB^2 + BC^2 + CA^2}{3}.
\]

8. Prove that the equation \( x^3 + y^3 = 7z^3 \) has infinitely many solutions in integers \( x, y, z \) with \( \gcd(x,y,z) = 1 \).