Italian IMO Team Selection Test 1997

Cortona, May 24, 1997

Time allowed: 4 hours

1. Let $\alpha,\beta,\gamma,\lambda$ be real numbers with α,β,γ not all equal such that
$$\alpha+\frac{1}{\beta}=\beta+\frac{1}{\gamma}=\gamma+\frac{1}{\alpha}=\lambda.$$
Find all possible values of λ and show that $\alpha\beta\gamma+\lambda=0$.

2. Let ABC be a triangle with $AB=AC$. Suppose that the bisector of $\angle ABC$ meets the side AC at point D such that $BC=BD+AD$. Find the measure of $\angle BAC$.

3. Determine all triples (x,y,p) with x,y positive integers and p a prime number verifying the equation $p^x-y^p=1$.

4. There are n pawns on n distinct squares of a 19×19 chessboard. In each move, all the pawns are simultaneously moved to a neighboring square (horizontally or vertically) so that no two are moved onto the same square. No pawn can be moved along the same line in two successive moves. What largest number of pawns can a player place on the board (being able to arrange them freely) so as to be able to continue the game indefinitely?

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com