

# Italian IMO Team Selection Test 1995

Cortona, 1995

Time allowed: 4 hours

1. Determine all triples  $(x, y, z)$  of integers greater than 1 with the property that  $x$  divides  $yz - 1$ ,  $y$  divides  $zx - 1$ , and  $z$  divides  $xy - 1$ .
2. Twenty-one rectangles of size  $3 \times 1$  are placed on an  $8 \times 8$  chessboard, leaving only one free unit square. What position can the free square lie at?
3. A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  verifies the conditions

$$\begin{cases} f(x+24) \leq f(x) + 24 \\ f(x+77) \geq f(x) + 77 \end{cases} \quad \text{for all } x \in \mathbb{R}.$$

Prove that  $f(x+1) = f(x) + 1$  for all real  $x$ .

4. In a triangle  $ABC$ ,  $P$  and  $Q$  are the feet of the altitudes from  $B$  and  $A$  respectively. Find the locus of the circumcenter of triangle  $PQC$ , when point  $C$  varies (with  $A$  and  $B$  fixed) in such a way that  $\angle ACB$  is equal to  $60^\circ$ .