10-th Italian Mathematical Olympiad 1994

May 6, 1994

1. Show that there exists an integer N such that for all $n \geq N$ a square can be partitioned into n smaller squares.

2. Find all integer solutions of the equation $y^2 = x^3 + 16$.

3. A journalist wants to report on the island of scoundrels and knights, where all inhabitants are either scoundrels (and they always lie) or knights (and they always tell the truth). The journalist interviews each inhabitant exactly once and gets the following answers:

   A_1: On this island there is at least one scoundrel;

   A_2: On this island there are at least two scoundrels;

   ...

   A_{n-1}: On this island there are at least $n-1$ scoundrels;

   A_n: On this island everybody is a scoundrel.

   Can the journalist decide whether there are more scoundrels or more knights?

4. Let ABC be a triangle contained in one of the halfplanes determined by a line r. Points A', B', C' are the reflections of A, B, C in r, respectively. Consider the line through A' parallel to BC, the line through B' paralle to AC and the line through C' parallel to AB. Show that these three lines have a common point.

5. Let OP be a diagonal of a unit cube. Find the minimum and the maximum value of the area of the intersection of the cube with a plane through OP.

6. The squares of a 10×10 chessboard are labelled with $1, 2, \dots, 100$ in the usual way: the $i-$th row contains the numbers $10i-9, 10i-8, \dots, 10i$ in increasing order. The signs of fifty numbers are changed so that each row and each column contains exactly five negative numbers. Show that after this change the sum of all numbers on the chessboard is zero.