

Grosman Memorial Mathematical Olympiad 1997

1. Prove that there are at most three primes between 10 and 10^{10} all of whose decimal digits are 1.
2. Is there a planar polygon whose vertices have integer coordinates and whose area is $1/2$, such that this polygon is
 - (a) a triangle with at least two sides longer than 1000?
 - (b) a triangle whose sides are all longer than 1000?
 - (c) a quadrangle?
3. Find all real solutions of $\sqrt[4]{13+x} + \sqrt[4]{4-x} = 3$.
4. Prove that if two altitudes of a tetrahedron intersect, then so do the other two altitudes.
5. Consider partitions of an $n \times n$ square (composed of n^2 unit squares) into rectangles with one integer side and the other side equal to 1. What is the largest possible number of such partitions among which no two have an identical rectangle at the same place?
6. In the plane are given $n^2 + 1$ points, no three of which lie on a line. Each line segment connecting a pair of these points is colored by either red or blue. A *path* of length k is a sequence of k segments where the end of each segment (except for the last one) is the beginning of the next one. A path is *simple* if it does not intersect itself. Prove that there exists a monochromatic simple path of length n .