15-th Iranian Mathematical Olympiad 1997/1998

First Round

Time: 3 hours each day.

First Day

1. Let x and y be positive integers such that $3x^2+x=4y^2+y$. Prove that $x-y$ is a square.

2. Let KL and KN be tangent to the circle $\mathscr{C}$ (with L,N on $\mathscr{C}$), and let M be a point on the extension of KN beyond N. The circumcircle of triangle KLM meets $\mathscr{C}$ again at P. Point Q is the foot of the perpendicular from N to ML. Prove that $\angle MPQ = 2\angle KML$.

3. An $n\times n$ table is filled with numbers $-1,0,1$ in such a manner that every row and column contains exactly one 1 and one -1. Prove that the rows and columns can be reordered so that in the resulting table each number has been replaced with its negative.

Second Day

4. Let x_1,x_2,x_3,x_4 be positive numbers with the product 1. Prove that
$$\sum_{i=1}^{4} x_i^3 \geq \max\left\{\sum_{i=1}^{4} x_i, \sum_{i=1}^{4}\frac{1}{x_i}\right\}.$$

5. In an acute triangle ABC, D is the foot of the altitude from A. The bisectors of the inner angles B and C respectively meet AD at E and F. If $BE=CF$, prove that ABC is an isosceles triangle.

6. Suppose a,b are natural numbers such that
$$p=\frac{b}{4}\sqrt{\frac{2a-b}{2a+b}}$$
is a prime number. What is the maximum possible value of p?

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