18-th Iranian Mathematical Olympiad 2000/01

First Round

Time: 4 hours each day.

First Day

1. Let S be a 21-element subset of $\{1, 2, \ldots, 2046\}$. Prove that there exist distinct numbers $a, b, c \in S$ such that $bc < 2a^2 < 4bc$.

2. Let D, E, F be points on the sides Bc, CA, AB respectively of a triangle ABC. Prove that the centroids of triangles ABC and DEF coincide if and only if $BD/DC = CE/EA = AF/FB$.

3. Prove that it is possible to choose 16 subsets of $M = \{1, 2, \ldots, 10000\}$ with the following property: For every $a \in M$, one can choose eight of the subsets whose intersection is $\{a\}$.

Second Day

4. Find all integers n for which the set $\{1, 2, \ldots, n\}$ can be written as the disjoint union of subsets A, B, C with equal sums of elements.

5. In tetrahedron $ABCD$, the sum of the angles at each vertex is $180°$. Prove that the faces of the tetrahedron are congruent triangles.

6. A *hyper-number* is a sequence $\overline{\ldots a_3 a_2 a_1}$, where each a_i is a decimal digit. In particular, every natural number can be regarded as a hyper-number with $a_i = 0$ for sufficiently large i. Hyper-numbers can be added and multiplied in a manner analogous to that for natural numbers.

   (a) Let A be a hyper-number. Prove that there is a hyper-number B such that $A + B = 0$.

   (b) Find all hyper-numbers A for which there is a hyper-number B such that $AB = 1$.

   (c) Is it true that $AB = 0$ implies $A = 0$ or $B = 0$? Justify your answer.

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