11-th Indian Mathematical Olympiad 1996

1. (a) Show that, for any positive integer n, there exist distinct positive integers x and y such that $x+j$ divides $y+j$ for $j=1,2,\dots,n$.

   (b) If for some positive integers x and y, $x+j$ divides $y+j$ for all positive integers j, show that $x=y$.

2. Let C_1 and C_2 be two concentric circles in the plane with radii R and $3R$ respectively. Show that the orthocenter of any triangle inscribed in C_1 lies in the interior of C_2. Conversely, show that every point in the interior of C_2 is the orthocenter of some triangle inscribed in C_1.

3. Solve in real numbers a,b,c,d,e the following system of equations:

$$3a=(b+c+d)^3, \quad 3b=(c+d+e)^3, \quad 3c=(d+e+a)^3,$$
$$3d=(e+a+b)^3, \quad 3e=(a+b+c)^3.$$

4. Find the number of ordered triples (A,B,C) of subsets of a given n-element set X such that $A\subset B\subsetneq C$.

5. The sequence $(a_n)_{n\in\mathbb{N}}$ is defined by $a_1=1$, $a_2=2$, and

$$a_{n+2}=2a_{n+1}-a_n+2 \quad \text{for } n\geq 1.$$

   Prove that for any m, a_ma_{m+1} is also a term of the sequence.

6. Given a $2n\times 2n$ array of 0's and 1's containing exactly $3n$ zeros, show that it is possible to remove all the zeros by deleting some n rows and n columns.

www.imomath.com

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
Typed in LaTeX by Ercole Suppa
www.imomath.com