

9-th Indian Mathematical Olympiad 1994

- In a triangle ABC with an obtuse angle at C , AD and CF are the medians and G the centroid.
 - If points B, D, G, F lie on a circle, show that $\frac{AC}{BC} \geq \sqrt{2}$.
 - Moreover, if P is the fourth vertex of the parallelogram $AGCP$, prove that triangle GAP is similar to $\triangle ABC$.
- Prove that if x is a real root of $x^5 - x^3 + x = a$, then $x^6 \geq 2a - 1$.
- Prove that among any 181 perfect squares there exist 19 whose sum is divisible by 19.
- Find the number of (nondegenerate) triangles whose vertices lie in the set of points (s, t) in the plane with $s, t \in \{0, 1, 2, 3, 4\}$.
- A circle through vertex C of a rectangle $ABCD$ is tangent to sides AB and AD at M and N . Given that the distance from C to the line MN equals 5, compute the area of rectangle $ABCD$.
- Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ which satisfy

$$f(-x) = f(x) \quad \text{and} \quad f(x+1) = f(x) + 1 \quad \text{for all } x, \text{ and}$$
$$f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2} \quad \text{for all } x \neq 0.$$