# 7-th Indian Mathematical Olympiad 1992

1. In a triangle $ABC$, $\angle A = 2\angle B$. Prove that $a^2 = b(b+c)$.

2. If real numbers $x, y, z$ satisfy $x+y+z=4$ and $x^2+y^2+z^2=6$, show that each of $x, y, z$ lies in the segment $\left[\frac{2}{3}, 2\right]$. Can $x$ attain either of the endpoints of the segment?

3. Determine the remainder of $19^{92}$ upon division by 92.

4. Find the number of permutations $(p_1, \dots, p_6)$ of $1, 2, \dots, 6$ such that for any $k$, $1 \le k \le 5$, $(p_1, \dots, p_k)$ does not form a permutation of $1, 2, \dots, k$.

5. Two circles $C_1$ and $C_2$ in the plane meet at points $P$ and $Q \neq P$. A line through $P$ meets $C_1$ at $A$ and $C_2$ at $B$. Let $Y$ be the midpoint of $AB$ and let $QY$ meet the cirlces $C_1$ and $C_2$ again at $X$ and $Z$ respectively. Show that $Y$ is the midpoint of $XZ$.

6. Let $f(x)$ be a polynomial with integer coefficients such that there exist distinct integers $a_1, \dots, a_5$ at which $f$ takes the value 2. Show that there does not exist an integer $b$ with $f(b) = 9$.

7. For each integer $n \ge 3$, find the number of ways in which one can place the numbers $1, 2, \dots, n^2$ in the squares of an $n \times n$ chessboard (one on each) such that the numbers in each row and in each column form an arithmetic progression.

8. Find all pairs $(m, n)$ of positive integers for which $2^m + 3^n$ is a perfect square.

9. Find $n$ such that in a regular $n$-gon $A_1A_2 \dots A_n$ we have
$$\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}.$$

10. Determine all functions $f : \mathbb{R} \setminus [0,1] \to \mathbb{R}$ such that for all $x$,
$$f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)}.$$



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