13-th Iberoamerican Mathematical Olympiad

Puerto Plata, Dominican Republic, September 19–27, 1998

First Day

1. On a circle are given 98 distinct points. Maria and José alternate drawing segments between two of the points which have not yet been connected by a segment. The game ends when each point is an endpoint of at least one segment. The winner is the player who draws the last segment. If José plays first, who can ensure a victory?

2. The incircle of a triangle ABC is tangent to BC, CA, AB at points D, E, F, respectively. Line AD meets the incircle again at Q. Prove that EQ passes through the midpoint of AF if and only if $AC = BC$.

3. Find the smallest number n with the following property: Among any n distinct numbers from the set $\{1, 2, \dots, 999\}$ there exist four numbers a, b, c, d with $a + 2b + 3c = d$.

Second Day

4. Representatives of $n \geq 2$ countries are sitting at a round table. If two representatives are from the same country, then their neighbors on the right are from two different countries. For each n, find the maximum possible number of representatives.

5. Find the largest n for which there exist different points $P_1, P_2, \dots, P_n$ in the plane and real numbers $r_1, r_2, \dots, r_n$ such that $P_iP_j = r_i + r_j$ for any $i \neq j$.

6. Let λ be the positive root of the equation $t^2 - 1998t - 1 = 0$. Define the sequence $(x_n)_{n\geq 0}$ by
$$x_0 = 1 \quad \text{and} \quad x_{n+1} = [\lambda x_n] \text{ for } n = 0, 1, 2, \dots$$
Determine the remainder of x_{1998} upon division by 1998.

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