10-th Iberoamerican Mathematical Olympiad

Valparaiso, Chile, September 23–30, 1995

First Day – September 26

1. Determine all possible values of the sum of the digits of a perfect square. *(Brazil)*

2. Given an integer $n > 1$, determine the real numbers $x_1, x_2, \dots, x_n \geq 1$ and $x_{n+1} > 0$, such that the following conditions are simultaneously fulfilled:

   (i) $\sqrt{x_1} + \sqrt[3]{x_2} + \cdots + \sqrt[n+1]{x_n} = n\sqrt{x_{n+1}}$,

   (ii) $\dfrac{x_1 + x_2 + \cdots + x_n}{n} = x_{n+1}$. *(Spain)*

3. Let r and s be two orthogonal lines, not belonging to the same plane. Let AB be their common perpendicular, with $A \in r$ and $B \in s$. The points $M \in r$ and $N \in s$ are variable so that MN is tangent to the sphere with diameter AB at some point T. Find the locus of T. *(Brazil)*

Second Day – September 27

4. Coins are situated on an $m \times m$ board. Each coin situated on the board is said to *dominate* all the cells of the row ($\rightarrow$), the column ($\downarrow$), and the diagonal ($\searrow$). Note that the coin does not dominate the diagonal ($\swarrow$). Determine the smallest number of coins which must be placed in order that all the cells of the board be dominated. *(Argentina)*

5. The incircle of a triangle ABC is tangent to BC, CA and AB at D, E and F, respectively. Suppose that the incircle passes through the midpoint X of AD. The lines XB and XC meet the incircle again at Y and Z, respectively. Show that $EY = FZ$. *(Spain)*

6. A function $f : \mathbb{N} \rightarrow \mathbb{N}$ is called *circular* if for each $p \in \mathbb{N}$ there exists $n \in \mathbb{N}$ with $n \leq p$ such that
$$f^n(p) = \underbrace{f(f(\dots f}_{p \text{ times}}(p))\dots) = p.$$
   The function f has *repulse degree* k, $0 < k < 1$, if for each $p \in \mathbb{N}$ we have $f^i(p) \neq p$ for all $0 < i \leq [kp]$. Determine the biggest repulse degree that can be reached by a circular function. *(Chile–Brazil)*

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