# 8-th Iberoamerican Mathematical Olympiad

## Mexico City, Mexico, September 11–19, 1993

*First Day – September 14*

1. Let $x_1 < x_2 < x_3 < \cdots$ be all the palindromic natural numbers, and for each $i$, define $y_i = x_{i+1} - x_i$. How many distinct primes belong to the set $\{y_i \mid i \in \mathbb{N}\}$? *(Argentina)*

2. Show that for any convex polygon of unit area, there exists a parallelogram of area 2 containing the polygon. *(Mexico)*

3. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that

   (i) if $x < y$, then $f(x) < f(y)$

   (ii) $f(yf(x)) = x^2 f(xy)$ for all $x, y \in \mathbb{N}$. *(Mexico)*

*Second Day – September 15*

4. Let $\Gamma$ be the incircle of an equilateral triangle $ABC$. If $D$ and $E$ are points on the sides $AB$ and $AC$, respectively, such that $DE$ is tangent to $\Gamma$, prove that
$$\frac{AD}{DB} + \frac{AE}{EC} = 1.$$
*(Spain)*

5. For any distinct points $P$ and $Q$ of the plane, we denote $m(PQ)$ the perpendicular bisector of the segment $PQ$. Let $S$ be a finite subset of the plane, with more than one element, which satisfies the following conditions:

   (i) If $P$ and $Q$ are distinct points of $S$, then $m(PQ)$ meets $S$.

   (ii) If $P_1Q_1$, $P_2Q_2$ and $P_3Q_3$ are three distinct segments with endpoints in $S$, then no point of $S$ belongs simultaneously to the three lines $m(P_1Q_1)$, $m(P_2Q_2)$, $m(P_3Q_3)$. *(Mexico)*

6. Two nonnegative integers $a$ and $b$ are *friends* if the decimal expression of $a+b$ is formed only by 0's and 1's. Suppose that $A$ and $B$ are two infinite sets of nonnegative integers such that $B$ is the set of all numbers which are friends of all the elements of $A$, $A$ is the set of all numbers which are friends of all the elements of $B$. Show that one of the sets $A, B$ contains infinitely many pairs of numbers $x, y$ with $x - y = 1$. *(Argentina)*



The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com