7-th Iberoamerican Mathematical Olympiad

Caracas, Venezuela, September 19–27, 1992

First Day

1. For each positive integer n, a_n denotes the last digit of $1+2+\cdots+n$. Evaluate $a_1+a_2+\cdots+a_{1992}$.

2. Let $a_1, a_2, \dots, a_n$ be positive numbers. Consider the function
$$f(x)=\frac{a_1}{x+a_1}+\frac{a_2}{x+a_2}+\cdots+\frac{a_n}{x+a_n}.$$
Determine the sum of the lengths of the intervals where $f(x)\geq 1$.

3. Circle G is inscribed in an equilateral triangle of side 2.
   (a) Prove that for every point P on G, $PA^2+PB^2+PC^2=5$.
   (b) Prove that for every P on G the segments PA, PB, PC are the sides of a triangle whose area is $\frac{\sqrt{3}}{4}$.

Second Day

4. The sequences of integers (a_n) and (b_n) have the following properties:
   (i) $a_0=0$, $b_0=8$;
   (ii) $a_{n+2}=2a_{n+1}-a_n+2$, $b_{n+2}=2b_{n+1}-b_n$ for $n>0$;
   (iii) $a_n^2+b_n^2$ is a square for all n.

   Find at least two possible values for (a_{1992}, b_{1992}).

5. We are given a circle C, the altitude h of a trapezoid $ABCD$ inscribed in C, and the sum m of the lengths of the bases AB and CD. Show how to construct the trapezoid $ABCD$.

6. A triangle ABC is given. Points $A_1, A_2, B_1, B_2, C_1, C_2$ are taken on the rays BA, CA, CB, AB, AC, BC respectively such that $AA_1=AA_2=BC$, $BB_1=BB_2=CA$, and $CC_1=CC_2=AB$. Prove that the area of the hexagon $A_1A_2B_1B_2C_1C_2$ is at least 13 times the area of the triangle ABC.

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