6-th Iberoamerican Mathematical Olympiad

Córdoba, Argentina, September 21–30, 1991

First Day

1. Each vertex of a cube is asigned +1 or -1, and each face is assigned the product of the numbers at its vertices. Which values can the sum of these 14 numbers take?

2. Two perpendicular lines divide a square into four parts, three of which have the area 1. Show that the area of the square is 4.

3. An increasing function f is defined for $0 \leq x \leq 1$ and satisfies:

   (a) $f(0) = 0$;

   (b) $f\left(\frac{x}{3}\right) = \frac{f(x)}{2}$;

   (c) $f(1-x) = 1 - f(x)$.

   Evaluate $f\left(\frac{18}{1991}\right)$.

Second Day

4. Determine a five-digit number N whose digits are nonzero and which is equal to the sum of all distinct three-digit numbers that can be formed from the digits of N.

5. Let $P(x,y) = 2x^2 - 6xy + 5y^2$. We say that an integer a is a *value* of P if there exist integers b, c for which $a = P(b,c)$.

   (a) How many elements of $\{1, 2, \ldots, 100\}$ are values of P?

   (b) Prove that a product of values of P is also a value of P.

6. Given three non-collinear points M, N, and P such that M and N are the midpoints of two sides of a certain triangle and P is the orthocenter of the triangle. Construct the triangle.

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