4-th Iberoamerican Mathematical Olympiad

La Habana, Cuba, April 8–16, 1989

First Day

1. Determine all triples (x,y,z) of real numbers satisfying the system of equations

$$\begin{aligned} x+y-z &= -1, \\ x^2-y^2+z^2 &= 1, \\ -x^3+y^3+z^3 &= -1. \end{aligned}$$

2. Let x,y,z be real numbers with $0 \le x,y,z \le \frac{\pi}{2}$. Prove the inequality

$$\frac{\pi}{2}+2\sin x\cos y+2\sin y\cos z \ge \sin 2x+\sin 2y+\sin 2z.$$

3. Let a,b and c be the side lengths of a triangle. Prove that

$$\frac{a-b}{a+b}+\frac{b-c}{b+c}+\frac{c-a}{c+a}<\frac{1}{16}.$$

Second Day

4. The incircle of the triangle ABC is tangent to sides AB and AC at M and N, respectively. The bisectors of the angles at A and B intersect MN at points P and Q, respectively. Let O be the incenter of $\triangle ABC$. Prove that $MP\cdot OA = BC\cdot OQ$.

5. A functions f is defined on the set $\mathbb{N}$ and satisfies

   (i) $f(1)=1$,

   (ii) $f(2n+1)=f(2n)+1$,

   (iii) $f(2n)=3f(n)$

   for all $n\in\mathbb{N}$. Find the set of values taken by f.

6. Show that the equation $2x^2-3x=3y^2$ has infinitely many solutions in positive integers.

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