2-nd Iberoamerican Mathematical Olympiad

Lima, Peru, April 24 – May 1, 1988

First Day

1. The sides of a triangle form an arithmetic progression, and so do the altitudes. Show that the triangle is equilateral.

2. The positive integers a,b,c,d,p, and q satisfy $ad-bc=1$ and $\frac{a}{b}>\frac{p}{q}>\frac{c}{d}$. Prove that:

   (a) $q \geq b+d$;

   (b) If $q=b+d$, then $p=a+c$.

3. Prove that, among all possible triangles whose vertices are 3, 5 and 7 apart from a given point P, the ones with the largest perimeter have P as incenter.

Second Day

4. Let ABC be a triangle with sides a,b,c. Each side of the triangle is divided into n equal parts. Denote by S the sum of the squares of the distances from the vertices to the division points on the opposite side. Prove that $\frac{S}{a^2+b^2+c^2}$ is a rational number.

5. Consider all the numbers of the form $x+yt+zt^2$, where x,y,z are rational numbers and $t=\sqrt[3]{2}$. Prove that if $x+yt+zt^2 \neq 0$, then there exist rational numbers u,v,w such that
$$(x+yt+zt^2)(u+vt+wt^2)=1.$$

6. consider all sets of n distinct positive integers, no three of which form an arithmetic progression. Prove that among all such sets there is one which has the largest sum of the reciprocals of its elements.

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