2-nd Iberoamerican Mathematical Olympiad

Salto – Paysandú, Uruguay, Januar 23 – February 1, 1987

First Day

1. Find the function $f(x)$ such that

$$f(x)^2 f\left(\frac{1-x}{1+x}\right) = 64x \quad \text{for all } x \notin \{0,-1,1\}.$$

2. In a triangle ABC, P is the centroid and M and N the midpoints of the sides AC and AB respectively. Prove that if a circle can be inscribed in the quadrilateral $ANPM$, then $\triangle ABC$ is isosceles.

3. Prove that if m,n,r are positive integers with r odd such that

$$\left(2+\sqrt{3}\right)^r = 1+m+n\sqrt{3},$$

then m is a perfect square.

Second Day

4. The sequenxce (p_n) is defined as follows: $p_1 = 2$ and for each $n \geq 2$, p_n is the greatest prime divisor of $p_1 p_2 \cdots p_{n-1} + 1$. Prove that every p_n is different from 5.

5. Let r,s,t be the roots of the equation $x(x-2)(3x-7) = 2$. Show that r,s,t are real and positive and determine $\arctan r + \arctan s + \arctan t$.

6. Let $ABCD$ be a convex quadrilateral and let P and Q be the points on the sides AD and BC respectively such that $AP/PD = BQ/QC = AB/CD$. Prove the line PQ forms equal angles with the lines AB and CD.

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