17-th Iberoamerican Mathematical Olympiad

San Salvador, El Salvador, September 28–October 6, 2002

First Day

1. The integers from 1 to 2002 are written in a sequence in this order. We erase the first, fourth, seventh term, etc, every term on the $3k+1$-th position ($k \in \mathbb{N}_0$), thus obtaining a new sequence. Then this procedure is repeated until only one number remains. Determine this number.

2. Given any set of nine points in the plane, no three of which are collinear, show that for any point P from this set the number of triangles having the vertices in this set and containing P in the interior is even.

3. A point P inside an equilateral triangle ABC satisfies $\angle APC = 120^\circ$. The rays CP and AP meet AB and BC respectively at points M and N. Find the locus of the circumcenter of triangle MBN when P assumes all possible positions.

Second Day

4. In a scalene triangle ABC, BD is an inner angle bisector with D on AC. Let E and F be the respective projections of A and C on the line BD, and let M be the projection of D on BC. Prove that $\angle EMD = \angle DMF$.

5. The sequence $(a_n)_{n=1}^{\infty}$ is defined by $a_1 = 56$ and

$$a_{n+1} = a_n - \frac{1}{a_n} \quad \text{for } n \geq 1.$$

   Show that $a_k < 0$ for some integer $k \leq 2002$.

6. A police is trying to capture a thief on a 2001×2001 board. They move alternately, and the player on turn moves to the adjacent square in one of the following three directions: $\downarrow$, $\rightarrow$, $\nwarrow$. Moreover, the police can move from the bottom right corner to the bottom left corner in one move (the thief cannot do this). Initially, the police is positioned in the central square and the thief is in the upper-left adjacent square. The police does the first move.

   (a) The thief can make at least 10000 moves before being captured.

   (b) The police has a strategy to capture the thief.

   Note: The police captures the thief if it enters the square occupied by the thief, but not if the thief enters the square occupied by the police.

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