15-th Iberoamerican Mathematical Olympiad

Caracas, Venezuela, September 16–24, 2000

First Day

1. The vertices of a regular n-gon have been labelled by 1 to n ($n \geq 3$). Show that, if n is odd, it is possible to assign to each side or diagonal an integer from 1 to n so that the following two conditions are fulfilled:

   (i) The number assigned to each side or diagonal is different from the labels of its vertices;

   (ii) For each vertex, the sides and diagonals emanating from it have different labels.

2. Two circles S_1 and S_2 with the respective centers O_1 and O_2 intersect at M and N. Their common tangent t, closer to M, touches S_1 at A and S_2 at B. Point C is diametrically opposite to B, and D is the intersection of line O_1O_2 with the perpendicular to AM from B. Prove that M, D and C are collinear.

3. Find all solutions of the equation $(x+1)^y - x^z = 1$ in integers greater than 1.

Second Day

4. Some terms of an infinite arithmetic progression $1, a_1, a_2, \dots$ of real numbers have been omitted, yielding an infinite geometric progression $1, b_1, b_2, \dots$ of common ratio q. Find all possible values of q.

5. There is a pile with 2000 stones. Two players alternately take the stones according to the following rules:

   (i) A player in turn takes 1,2,3,4 or 5 stones from the pile;

   (ii) It is prohibited to take as many stones as the opponent did in the previous move.

   The player who cannot perform a legal move loses the game. Decide which player has a winning strategy.

6. A convex hexagon is called *nice* if it has four diagonals of length 1 whose endpoints include all the vertices of the hexagon.

   (a) For every $0 < k < 1$, give an example of a nice hexagon of area k.

   (b) Prove that the area of a nice hexagon is less than $3/2$.

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