1. Let \( n \) be a positive integer. Each point \((x, y)\) in the plane, where \( x \) and \( y \) are nonnegative integers with \( x + y = n \), is colored red or blue, subject to the following condition: If a point \((x, y)\) is red, then so are all points \((x', y')\) with \( x' \leq x \) and \( y' \leq y \). Let \( A \) be the number of ways to choose \( n \) blue points with distinct \( x \)-coordinates, and let \( B \) be the number of ways to choose \( n \) blue points with distinct \( y \)-coordinates. Prove that \( A = B \).  

(Colombia)

2. The circle \( S \) has center \( O \), and \( BC \) is a diameter of \( S \). Let \( A \) be a point of \( S \) such that \( \angle AOB < 120^\circ \). Let \( D \) be the midpoint of the arc \( AB \) that does not contain \( C \). The line through \( O \) parallel to \( DA \) meets the line \( AC \) at \( I \). The perpendicular bisector of \( OA \) meets \( S \) at \( E \) and at \( F \). Prove that \( I \) is the incenter of the triangle \( CEF \).  

(South Korea)

3. Find all pairs of positive integers \( m, n \geq 3 \) for which there exist infinitely many positive integers \( a \) such that  
\[
\frac{a^n + a - 1}{a^n + a^2 - 1}
\]
is itself an integer.  

(Romania)

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4. Let \( n \geq 2 \) be a positive integer, with divisors \( 1 = d_1 < d_2 < \cdots < d_k = n \). Prove that \( d_1 d_2 + d_2 d_3 + \cdots + d_{k-1} d_k \) is always less than \( n^2 \), and determine when it is a divisor of \( n^2 \).  

(Romania)

5. Find all functions \( f \) from the reals to the reals such that  
\[
(f(x) + f(z))(f(y) + f(t)) = f(xy - zt) + f(xt + yz)
\]
for all real \( x, y, z, t \).  

(India)

6. Let \( n \geq 3 \) be a positive integer. Let \( C_1, C_2, C_3, \ldots, C_n \) be unit circles in the plane, with centers \( O_1, O_2, O_3, \ldots, O_n \) respectively. If no line meets more than two of the circles, prove that  
\[
\sum_{1 \leq i < j \leq n} \frac{1}{O_i O_j} \leq \frac{(n - 1) \pi}{4}.
\]

(Ukraine)