33-rd International Mathematical Olympiad

Moscow, Russia, July 10–21, 1992

First Day – July 15

1. Find all integer triples (p,q,r) such that $1 < p < q < r$ and $(p-1)(q-1)(r-1)$ is a divisor of $(pqr-1)$. *(New Zealand)*

2. Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that
$$f(x^2+f(y)) = y + f(x)^2 \text{ for all } x,y \text{ in } \mathbb{R}.$$ *(India)*

3. Given nine points in space, no four of which are coplanar, find the minimal natural number n such that for any coloring with red or blue of n edges drawn between these nine points there always exists a triangle having all edges of the same color. *(China)*

Second Day – July 16

4. In the plane, let there be given a circle C, a line l tangent to C, and a point M on l. Find the locus of points P that has the following property: There exist two points Q and R on l such that M is the midpoint of QR and C is the incircle of PQR. *(France)*

5. Let V be a finite subset of Euclidean space consisting of points (x,y,z) with integer coordinates. Let S_1,S_2,S_3 be the projections of V onto the yz,xz,xy planes, respectively. Prove that
$$|V|^2 \le |S_1||S_2||S_3|$$
($|X|$ denotes the number of elements of X). *(Italy)*

6. For each positive integer n, denote by $s(n)$ the greatest integer such that for all positive integer $k \le s(n)$, n^2 can be expressed as a sum of squares of k positive integers.
   (a) Prove that $s(n) \le n^2 - 14$ for all $n \ge 4$.
   (b) Find a number n such that $s(n) = n^2 - 14$.
   (c) Prove that there exist infinitely many positive integers n such that $s(n) = n^2 - 14$. *(Great Britain)*

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