8-th International Mathematical Olympiad

Sofia, Bulgaria, July 3–13, 1966

First Day

1. Three problems A, B, and C were given on a mathematics olympiad. All 25 students solved at least one of these problems. The number of students who solved B and not A is twice the number of students who solved C and not A. The number of students who solved only A is greater by 1 than the number of students who along with A solved at least one other problem. Among the students who solved only one problem, half solved A. How many students solved only B? *(Soviet Union)*

2. If a, b, and c are the sides and α, β, and γ the respective angles of the triangle for which $a+b=\tan\frac{\gamma}{2}(a\tan\alpha+b\tan\beta)$, prove that the triangle is isosceles. *(Hungary)*

3. Prove that the sum of distances from the center of the circumsphere of the regular tetrahedron to its four vertices is less than the sum of distances from any other point to the four vertices. *(Bulgaria)*

Second Day

4. Prove the following equality:

$$\frac{1}{\sin 2x}+\frac{1}{\sin 4x}+\frac{1}{\sin 8x}+\cdots+\frac{1}{\sin 2^n x}=\cot x-\cot 2^n x,$$

where $n \in \mathbb{N}$ and $x \notin \pi\mathbb{Z}/2^k$ for every $k \in \mathbb{N}$. *(Yugoslavia)*

5. Solve the following system of equations:

$$\begin{aligned}
|a_1-a_2|x_2+|a_1-a_3|x_3+|a_1-a_4|x_4&=1,\\
|a_2-a_1|x_1+|a_2-a_3|x_3+|a_2-a_4|x_4&=1,\\
|a_3-a_1|x_1+|a_3-a_2|x_2+|a_3-a_4|x_4&=1,\\
|a_4-a_1|x_1+|a_4-a_2|x_2+|a_4-a_3|x_3&=1,
\end{aligned}$$

where a_1, a_2, a_3, and a_4 are mutually distinct real numbers. *(Czechoslovakia)*

6. Let M, K, and L be points on (AB), (BC), and (CA), respectively. Prove that the area of at least one of the three triangles $\triangle MAL$, $\triangle KBM$, and $\triangle LCK$ is less than or equal to one-fourth the area of $\triangle ABC$. *(Poland)*

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