3-rd International Mathematical Olympiad

Budapest – Veszprem, Hungary, July 6–16, 1961

First Day

1. Solve the following system of equations:

$$\begin{aligned} x+y+z &= a, \\ x^2+y^2+z^2 &= b^2, \\ xy &= z^2, \end{aligned}$$

where a and b are given real numbers. What conditions must hold on a and b for the solutions to be positive and distinct? *(Hungary)*

2. Let a, b, and c be the lengths of a triangle whose area is S. Prove that

$$a^2+b^2+c^2 \geq 4S\sqrt{3}\,.$$

In what case does equality hold? *(Poland)*

3. Solve the equation $\cos^n x - \sin^n x = 1$, where n is a given positive integer. *(Bulgaria)*

Second Day

4. In the interior of $\triangle P_1P_2P_3$ a point P is given. Let Q_1, Q_2, and Q_3 respectively be the intersections of PP_1, PP_2, and PP_3 with the opposing edges of $\triangle P_1P_2P_3$. Prove that among the ratios PP_1/PQ_1, PP_2/PQ_2, and PP_3/PQ_3 there exists at least one not larger than 2 and at least one not smaller than 2. *(DR Germany)*

5. Construct a triangle ABC if the following elements are given: $AC = b$, $AB = c$, and $\measuredangle AMB = \omega$ ($\omega < 90^o$), where M is the midpoint of BC. Prove that the construction has a solution if and only if

$$b\tan\frac{\omega}{2} \leq c < b\,.$$

In what case does equality hold? *(Czechoslovakia)*

6. A plane ε is given and on one side of the plane three noncollinear points A, B, and C such that the plane determined by them is not parallel to ε. Three arbitrary points A', B', and C' in ε are selected. Let L, M, and N be the midpoints of AA', BB', and CC', and G the centroid of $\triangle LMN$. Find the locus of all points obtained for G as A', B', and C' are varied (independently of each other) across ε. *(Romania)*

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