1-st International Mathematical Olympiad

Bucharest – Brasov, Romania, July 23–31, 1959

First Day

1. For every integer n prove that the fraction $\frac{21n+4}{14n+3}$ cannot be reduced any further. *(Poland)*

2. For which real numbers x do the following equations hold:

$$\begin{aligned} &\text{(a)} \quad \sqrt{x+\sqrt{2x-1}}+\sqrt{x+\sqrt{2x-1}}=\sqrt{2}\ , \\ &\text{(b)} \quad \sqrt{x+\sqrt{2x-1}}+\sqrt{x+\sqrt{2x-1}}=1\ , \\ &\text{(c)} \quad \sqrt{x+\sqrt{2x-1}}+\sqrt{x+\sqrt{2x-1}}=2\ ? \end{aligned}$$

(Romania)

3. Let x be an angle and let the real numbers a, b, c, $\cos x$ satisfy the following equation:

$$a\cos^2 x + b\cos x + c = 0\ .$$

Write the analogous quadratic equation for a, b, c, $\cos 2x$. Compare the given and the obtained equality for $a = 4$, $b = 2$, $c = -1$. *(Hungary)*

Second Day

4. Construct a right-angled triangle whose hypotenuse c is given if it is known that the median from the right angle equals the geometric mean of the remaining two sides of the triangle. *(Hungary)*

5. A segment AB is given and on it a point M. On the same side of AB squares $AMCD$ and $BMFE$ are constructed. The circumcircles of the two squares, whose centers are P and Q, intersect in M and another point N.

   (a) Prove that lines FA and BC intersect at N.

   (b) Prove that all such constructed lines MN pass through the same point S, regardless of the selection of M.

   (c) Find the locus of the midpoints of all segments PQ, as M varies along the segment AB. *(Romania)*

6. Let α and β be two planes intersecting at a line p. In α a point A is given and in β a point C is given, neither of which lies on p. Construct B in α and D in β such that $ABCD$ is an equilateral trapezoid, $AB \parallel CD$, in which a circle can be inscribed. *(Czechoslovakia)*

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