Hungarian Mathematical Olympiad 1998/99

Final Round

Grades 11 and 12

1. Find all solutions $0 < x, y, z \leq 1$ of the equation
$$\frac{x}{1+y+zx} + \frac{y}{1+z+xy} + \frac{z}{1+x+yz} = \frac{3}{x+y+z}.$$

2. The midpoints of the edges of a tetrahedron are on a unit sphere. What is the maximum possible volume of the tetrahedron?

3. Positive integers are written in the fields of an $n^2 \times n^2$ chessboard such that the difference of any two edge neighbors is at most n. Prove that there is a number that occurs in at least $[n/2] + 1$ fields.

Grades 11 and 12 – specialized math classes

1. Let $n > 1$ be a real number and k be the number of positive primes not exceeding n. Suppose that $k+1$ positive integers are taken such that none of them divides the product of the others. Prove that one of these $k+1$ integers is greater than n.

2. The polynomial $x^4 - 2x^2 + ax + b$ has four distinct real roots. Show that the absolute value of every root is smaller than $\sqrt{3}$.

3. Each side of a convex polygon has an integer length, and the perimeter is odd. Prove that its area is at least $\sqrt{3}/4$.

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