Hungarian Mathematical Olympiad 2001/02

Final Round

Grades 11 and 12 – technical schools

1. The perpendicular from vertex A of the rectangle $ABCD$ to diagonal BD meets BD and side BC at the interior points E and F, respectively. Determine the ratio of the adjacent sides of $ABCD$ if segment EF subtends an angle of 30° at C.

2. Consider the following 2000 equations:

$$\begin{aligned} 1x^2 + 2 \cdot 2x + 3 &= 0 \\ 2x^2 + 2 \cdot 3x + 4 &= 0 \\ 3x^2 + 2 \cdot 4x + 5 &= 0 \\ &\vdots \\ 2000x^2 + 2 \cdot 2001x + 2002 &= 0. \end{aligned}$$

   For each equation, consider the product of the sum of the real roots and the sum of their reciprocals (if it exists). What is the product of these products?

3. Consider the set R of points with integer coordinates in the plane. A subset S of R has the property that the midpoint of a segment connecting any two points in S is not in R. At most how many elements does S contain?

Grades 11 and 12 – high schools

1. The excircle of triangle ABC at side AB touches AB at P and the line AC at Q. The excircle at side BC touches the extensions of sides AC and AB at U and X respectively. Prove that the intersection point of PQ and UX is equidistant from the lines AB and BC.

2. Is there an n-gon with exactly $n^2 - 30n + 236$ acute angles?

3. If $n \geq 1$ is a fixed integer, give an example of real numbers $x_1, x_2, \dots, x_n$ satisfying the equations

$$x_1 + x_2 + \cdots + x_n = 2(n-1),$$
$$(x_1 - 1)^2 + (x_2 - 1)^2 + \cdots + (x_n - 1)^2 = n$$

   with the maximum possible value of x_n.

www.imo.org.yu

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imo.org.yu

Grades 11 and 12 – specialized math classes

1. The real numbers $x_1, x_2, \ldots, x_n$ ($n \geq 2$) are such that each number equals the sum of the squares of the others. Find all such numbers $x_1, \ldots, x_n$.

2. An interior point is marked on each side of a parallelogram. Prove that the perimeter of the quadrilateral determined by these points is at least twice the length of the shorter diagonal.

3. Find a subset H of the positive integers with the following properties:

   (i) All sufficiently large positive integers can be written as the sum of at most 100 (not necessarily different) elements of H.

   (ii) 2002 is the least k for which all sufficiently large positive integers can be written as the sum of exactly k (not necessarily different) elements of H.

www.imo.org.yu

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imo.org.yu