Hungarian Mathematical Olympiad 2000/01

Final Round

Grades 11 and 12

1. Let S denote the number of 77-element subsets of $H = \{1, 2, \ldots, 2001\}$ with an even sum of elements, and N be the number of those with an odd sum. Decide whether S or N is greater, and by how much.

2. The base of a right pyramid is a regular hexagon $ABCDEF$ and the top vertex is P. The angle between the base and a face is equal to the angle between the faces ABP and CDP. Find the angle between the edge AP and the base.

3. Prove that for any positive numbers $a_1, a_2, \ldots, a_n$,
$$\frac{a_1^2}{a_1 + a_2} + \frac{a_2^2}{a_2 + a_3} + \cdots + \frac{a_n^2}{a_n + a_1} \geq \frac{1}{2}(a_1 + a_2 + \cdots + a_n).$$

Grades 11 and 12 – specialized math classes

1. For a positive integer c, let c_1, c_3, c_7, and c_9 denote the number of divisors of c ending with the digit 1,3,7, and 9, respectively (in the decimal system). Prove that $c_3 + c_7 \leq c_1 + c_9$.

2. Circles k_1, k_2 and a point P are given on the plane. Construct (if possible) a line through P which meets the circle k_i at A_i and B_i $(i = 1, 2)$ in such a way that there exist points C_i on k_i $(i = 1, 2)$ with $A_1C_1 = B_1C_1 = A_2C_2 = B_2C_2$.

3. Let $a_1, \ldots, a_k, b_1, \ldots, b_m$ be integers greater than 1. Every a_i is the product of an even number of (not necessarily distinct) primes, while every b_i is the product of an odd number of (not necessarily distinct) primes. In how many ways can we select a few numbers (maybe none or all) out of these $k + m$ integers in such a way that every b_i has an even number of divisors among the chosen numbers?

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