Hungarian Mathematical Olympiad 1999/2000

Final Round

Grades 11 and 12

1. Let H be a set of 2000 nonzero real numbers. How many negative elements should H have in order to maximize the number of four-element subsets of H with a negative product of elements?

2. Let C_1, A_1, B_1 be the midpoints of the sides AB, BC, CA of a scalene triangle ABC, respectively. Let A_2 be the midpoint of the polygonal line path BAC. Points B_2 and C_2 are analogously defined. Prove that the segments A_1A_2, B_1B_2, and C_1C_2 are concurrent.

3. Suppose that $\{a_1, a_2, \dots, a_n\}$ and $\{b_1, b_2, \dots, b_k\}$ are two sets of positive integers with the same sum of elements, and that this sum is less than nk. Show that it is possible to omit some (but not all) summands from both sides of the equality $a_1 + a_2 + \cdots + a_n = b_1 + b_2 + \cdots + b_k$ in such a way that the equality remains valid.

Grades 11 and 12 – specialized math classes

1. Show that on the graph of a polynomial function of an odd degree with integer coefficients there are no infinitely many points with integer coordinates whose pairwise distances are all integers.

2. Consider the circle passing through the intersection points of the angle bisectors of a given triangle with the opposite sides. Prove that this circle determines three chords on the lines of the sides of the triangle, the length of one of which equals the sum of the other two. (A chord may have the length 0.)

3. Let $k, t \geq 1$ be corpime integers. Starting with the numbers $1, 2, \dots, n$ in the increasing order, one can change places of any two numbers that differ by k or t. Show that every permutation can be obtained by repeating this process if and only if $n \geq k + t - 1$.

www.imo.org.yu

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
Typed in LaTeX by Ercole Suppa
www.imo.org.yu