

Eötvös Mathematical Competition 1896

1. If k is the number of distinct prime divisors of a natural number n , prove that $\log n \geq k \log 2$.
2. Prove that the equations

$$x^2 - 3xy + 2y^2 + x - y = 0 \quad \text{and} \quad x^2 - 2xy + y^2 - 5x + 7y = 0$$

imply the equation $xy - 12x + 15y = 0$.

3. Construct a triangle, given the feet of its altitudes. Express the sides of a triangle Y in terms of the sides of the triangle X formed by the feet of the altitudes of Y .