

# Kürschák Mathematical Competition 1985

1. Writing down the first 4 rows of the Pascal triangle in the usual way and then adding up the numbers in vertical columns, we obtain 7 numbers as shown below. If the same is done with the first 1024 rows of the Pascal triangle, how many of the 2047 numbers thus obtained will be odd?

$$\begin{array}{cccc} & & 1 & & & & \\ & & & 1 & & 1 & \\ & & & & 1 & & 1 \\ & 1 & & 2 & & 1 & \\ \hline 1 & 3 & & 3 & & 1 & \\ 1 & 1 & 4 & 3 & 4 & 1 & 1 \end{array}$$

2. Let  $A_1B_1A_2, B_1A_2B_2, A_2B_2A_3, \dots, B_{13}A_{14}B_{14}, A_{14}B_{14}A_1$  and  $B_{14}A_1B_1$  be equilateral rigid plates that can be folded along the edges  $A_1B_1, B_1A_2, \dots, A_{14}B_{14}$  and  $B_{14}A_1$  respectively. Can they be folded so that all 28 plates lie in the same plane?
3. Given are  $n$  integers, not necessarily distinct, and two positive integers  $p$  and  $q$ . If the  $n$  numbers are not all distinct, choose two equal ones. Add  $p$  to one of them and subtract  $q$  from the other. If there are still equal ones among the  $n$  numbers, repeat this procedure. Prove that after a finite number of steps all  $n$  numbers will be distinct.