Kürschák Mathematical Competition 1984

1. Rational numbers x, y and z satisfy the equation

$$x^3 + 3y^3 + 9z^3 - 9xyz = 0.$$

   Prove that $x = y = z = 0$.

2. If the polynomial $f(x) = x^n + a_1 x_{n-1} + \cdots + a_{n-1} x + 1$ has non-negative coefficients and n real roots, prove that $f(2) \geq 3^n$.

3. Given are $n+1$ points $P_1, P_2, \ldots, P_n$ and Q in the plane, no three collinear. Assume that for any two different points P_i and P_j there is a point P_k such that the point Q lies inside the triangle $P_i P_j P_k$. Prove that n is an odd number.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
Typed in LaTeX by Ercole Suppa
www.imomath.com