Kürschák Mathematical Competition 1982

1. For any five points A, B, P, Q, R in a plane, prove that
$$AB + PQ + QR + RP \leq AP + AQ + AR + BP + BQ + BR.$$

2. Let $n > 2$ be an even number. The squares of an $n \times n$ chessboard are colored with $\frac{1}{2}n^2$ colors in such a way that every color is used for coloring exactly two squares. Prove that one can place n rooks on squares of n different colors such that no two rooks attack each other.

3. For a positive integer n denote by $r(n)$ the sum of the remainders when n is divided by $1, 2, \ldots, n$ respectively. Prove that $r(k) = r(k-1)$ for infinitely many positive integers k.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
Typed in LaTeX by Ercole Suppa
www.imomath.com