Eötvös Mathematical Competition 1939

1. Let $a_1, a_2, b_1, b_2, c_1, c_2$ be real numbers for which $a_1a_2 > 0$, $a_1c_1 \geq b_1^2$ and $a_2c_2 > b_2^2$. Prove that

$$(a_1 + a_2)(c_1 + c_2) \geq (b_1 + b_2)^2.$$

2. Determine the highest power of 2 that divides $(2^n)!$.

3. In an acute triangle ABC, three semicircles are constructed outwardly on the sides BC, CA, and AB. Construct points A', B' and C' on the semicircles corresponding to A, B, C respectively such that $AB' = AC'$, $BC' = BA'$ and $CA' = CB'$.

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