Eötvös Mathematical Competition 1938

1. Prove that an integer n has a representation as a sum of two squares if and only if so does $2n$.

2. Prove that for all integers $n > 1$,
$$\frac{1}{n} + \frac{1}{n+1} + \cdots \frac{1}{n^2-1} + \frac{1}{n^2} > 1$$

3. Prove that for any acute triangle there is a point in space such that every segment joining a vertex to a point on the line through the other two vertices subtends a right angle at this point.

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