Eötvös Mathematical Competition 1937

1. Let $a_1, a_2, \ldots, a_n$ be positive integers and k be an integer greater than the sum of the a_i's. Prove that $a_1!a_2!\cdots a_n! < k!$.

2. Two circles in space are said to be tangent to each other if they have a common tangent at the same point of tangency. Assume that some three circles in space are mutually tangent at three distinct points. Prove that they either all lie in a plane or all lie on a sphere.

3. Let $P, Q, A_1, A_2, \ldots, A_n$ be distinct points such that the A_i are not collinear. Suppose that

$$PA_1 + PA_2 + \cdots + PA_n = QA_1 + QA_2 + \cdots + QA_n = s.$$

   Prove that there exists a point R such that $RA_1 + RA_2 + \cdots + RA_n < s$.

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