Eötvös Mathematical Competition 1936

1. Prove that for all positive integers n,

$$\frac{1}{1\cdot 2}+\frac{1}{3\cdot 4}+\cdots\frac{1}{(2n-1)2n}=\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}$$

2. A point S inside a triangle ABC is such that the areas of the triangles ABS, BCS and CAS are all equal. Prove that S is the centroid of $\triangle ABC$.

3. Let a be any positive integer. Prove that there exists a unique pair of positive integers (x,y) such that

$$x+\frac{1}{2}(x+y-1)(x+y-2)=a.$$

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com
Typed in LaTeX by Ercole Suppa