

Eötvös Mathematical Competition 1907

1. If p and q are odd integers, prove that the equation $x^2 + 2px + 2q = 0$ has no rational roots.
2. Let P be a point inside the parallelogram $ABCD$ and let R be the radius of the circle through A , B and C . Show that the distance from P to the nearest vertex is not greater than R .
3. Let $\frac{r}{s} = 0.k_1k_2k_3\cdots$ be the decimal expansion of a rational number. (If this is a terminating decimal, all k_i from a certain one on are 0.) Prove that at least two of the numbers

$$\sigma_i = 10^i \frac{r}{s} - (10^{i-1}k_1 + 10^{i-2}k_2 + \cdots + k_i)$$

are equal.